NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, APPLIED SCIENCES, AND NATURAL RESOURCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

QUALIFICATION : BACHELOR OF SCIENCE	
QUALIFICATION CODE: 07BOSC	LEVEL: 5
COURSE: PROBABILITY THEORY 1	COURSE CODE: PBT501S
DATE: NOVEMBER 2022	SESSION: NOVEMBER
DURATION: 3 HOURS	MARKS: 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	Dr. D. Ntirampeba
MODERATOR:	Mr. J. Amunyela

THIS QUESTION PAPER CONSISTS OF 4 PAGES
(Excluding this front page and statistical tables)

INSTRUCTIONS

1. Answer ALL the questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

1. Non-programable calculator

ATTACHMENTS
1. Statistical tables (Z-Table)

QUESTION 1 [40 Marks]

1.1. Define the following terminologies as they are applied in set theory and probability theory

1.1.1   A random experiment [2]

1.1.2   A sample space [2]

1.1.3   The size of a set A [2]

1.1.4   Pairwise mutually exclusive events [2]

1.1.5   Independent events (say A and B) [2]

1.2.   The probability mass function of the discrete random variable X is given by

$$P(x) = \begin{cases} c\binom{4}{x}\binom{5}{5-x}, & x = 0,1,2,3,4 \\ 0, & elsewhere \end{cases}$$

1.2.1.   Find the value of c [2]

1.2.2.   Assuming $c = \frac{1}{126}$, find the cumulative distribution of X [4]

1.2.3.   Find the median [1]

1.3.   An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} kx^{-4}, & x > 1 \\ 0, & otherwise \end{cases}$$

1.3.1.   Find the value of k so that $f(x)$ is a probability density function. [3]

1.3.2.   Assuming the value of k is 3, find:

1.3.2.1. the expected number of X [3]

1.3.2.2. the variance of number of X [4]

1.3.2.3. What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers [3]

1.4.	The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by.

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

Work out the following.

1.4.1.	standard deviation of the number of imperfections	[3]

1.4.2. $E(X^2 + 3)$	[2]

1.4.3 $Var(X^2 + 3)$	[5]

QUESTION 2 [20 Marks]

2.1.	At Hopewell Electronics, all 140 employees were asked about their political affiliations. The employees were grouped by type of work, as executives or production workers. The results are shown in the table below.

	Political affiliation		
Employee type	Democrat	Republican	Independent
Executive	5	34	9
Production worker	63	21	8

Suppose an employee is selected at random from Hopewell employees. Let us use the following notations to represent different events of choosing:

E =executive; W =production worker; D =democrat; R =republican; and I =independent

2.1.1	Compute $P(R)$	[1]

2.1.2	Compute $P(D \text{ and } E)$	[2]

2.1.3	Compute $P(W \text{ or } I)$	[2]

2.1.4	Compute $P(E|D)$	[3]

2.2.

A paint-store chain produces and sells latex and semigloss paint. Based on long-range sales, the probability that a customer will purchase latex paint is 0.75. Of those that purchase latex paint, 60% also purchase rollers. But only 30% of semigloss paint buyers purchase rollers. A randomly selected buyer purchases a roller and a can of paint. What is the probability that the paint is latex? [5]

2.3.

Customers who purchase a certain make of car can order an engine in any of three sizes. Of all cars sold, 45% have smallest engine, 35% have the medium size one, and 20% have largest. Of cars with small engine, 10% fail an emissions test within two years of purchase, while 12% of those with medium size and 15% of those with largest engine fail. A record for a failed emissions test is chosen at random.

2.3.1 What is probability that it is for a car with medium-size engine? [4]

2.3.2. What is probability that it is for a car with medium-size engine or largest engine? [3]

QUESTION 3 [20 Marks]

3.1. A soft-drink machine is regulated so that it discharges an average of 200 milliliters per cup. If the amount of drink is normally distributed with a standard deviation equal to 15 milliliters,

3.1.1. what fraction of the cups will contain more than 224 milliliters? [4]

3.1.2. below what value do we get the smallest 25% of the drinks? [5]

3.2. In a certain city district, the need for money to buy drugs is stated as the reason for 75% of all thefts.

3.2.1. Find the probability that among the next 5 theft cases reported in this district, at most 3 resulted from the need for money to buy drugs. [4]

3.2.2. What is the expected number of theft cases resulting from the need for money to buy drugs if 10 theft cases are observed? [2]

3.3. A secretary makes 2 errors per page, on average.

3.3.1. What is the probability that on the page he or she will make 4 or more errors? [4]

3.4.2. What is the expected number of errors in the next three pages? [1]

QUESTION 4[20 Marks]

4.1. A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshev's theorem find

$P(5 < X < 15)$ [5]

4.2 The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes.

4.2.1. What is the probability that a person is served in less than 3 minutes? [5]

4.2.2. Find the variance of the length of time for an individual to be served at the cafeteria. [2]

4. 3.

4.3.1. Let X be continuous uniform random variable on the interval [a, b]. show that

$E(X) = \frac{a+b}{2}$ [4]

4.3.2. The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable X having a continuous uniform distribution in interval $[7, 10]$. Find the probability that on a given day the amount of coffee dispensed by this machine will be at most 8.8 liters? [4]

END OF EXAMINATION QUESTION PAPER

Standard Normal Probabilities

Table entry

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

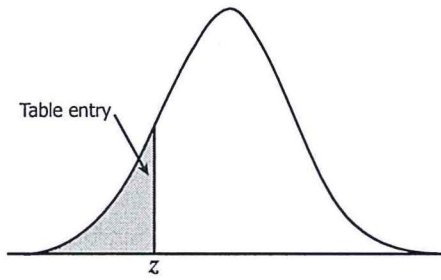# Standard Normal Probabilities

Table entry

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998